# Halo Mass Function & Halo Structure — ΛCDM vs FDM

Press–Schechter mass function and solitonic-core halo structure for fuzzy dark matter, $m = 8\times10^{-23}$ eV ($m_{22}$=0.8) and $10^{-22}$ eV ($m_{22}$=1.0).

`FDM reproduction campaign · response to Sandro's Task 1 & Task 2 · July 2026 · semi-analytic, Planck-2018 cosmology`

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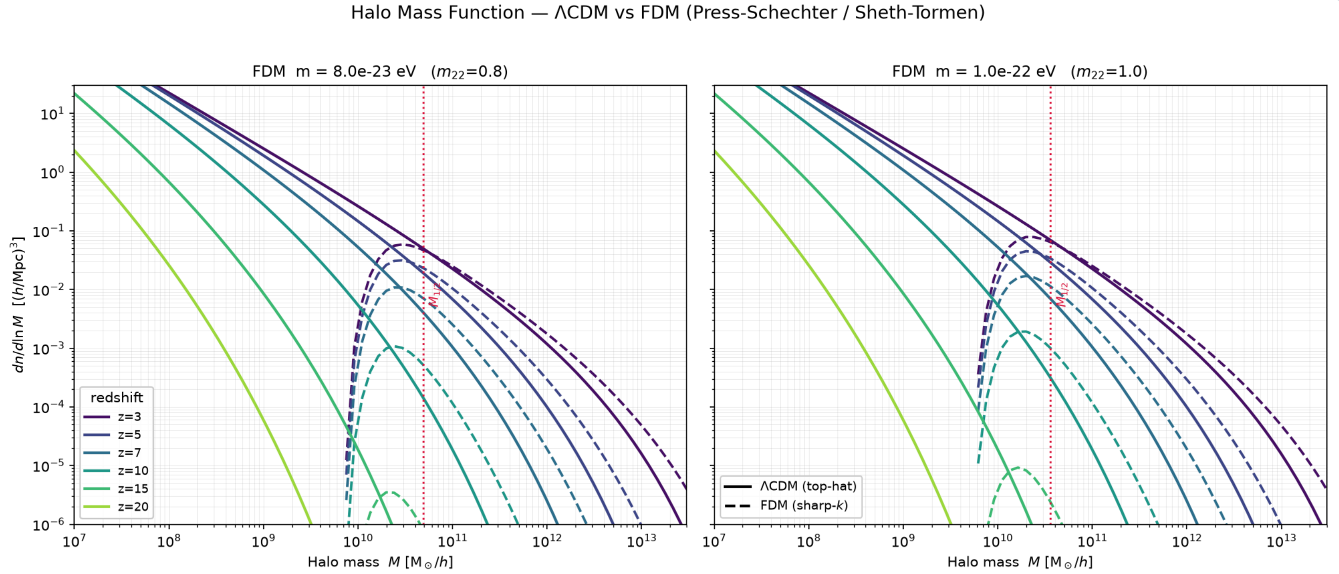
This memo answers two questions from Sandro on fuzzy dark matter, ΛCDM vs FDM: the halo mass function and halo structure. **Each question is stated verbatim at the head of its section**, followed by method, result, and validation. Results are semi-analytic (the named Press–Schechter method), with our GAMER / GADGET-4 simulation data overlaid where available — each figure carries a bold label saying whether it is theory or our simulation.

## 1 · Halo Mass Function (Task 1)

> **QUESTION 1 · SANDRO**
>
> **How many dark matter halos of which mass are forming in ΛCDM vs FDM? Plot the halo mass function as a function of redshift (z = 3–20) and halo mass.**

## Method

Press–Schechter / Sheth–Tormen abundance on our in-house, validated linear power spectrum: Eisenstein–Hu (1998) CDM transfer × Hu–Barkana–Gruzinov (2000) FDM cutoff, $\sigma_8$-normalized and growth-scaled. $\Lambda$CDM uses a real-space top-hat window; FDM uses a **sharp-k window** (mandatory — a top-hat integrates power the FDM cutoff has removed and manufactures spurious sub-cutoff halos). The FDM transfer is truncated at its first acoustic node; the sub-node oscillations of the fitting formula are not physical structure. The $\Lambda$CDM baseline reproduces **colossus** (Sheth–Tormen) to 0.1% (z=3–7), 5% (z=15).



EH98×Hu-Barkana-Gruzinov(2000) linear P(k); ΛCDM validated vs colossus. FDM cut off below the half-mode mass; suppression deepens toward high *z*.

ANALYTIC — FROM THEORY · NO SIMULATION DATA

**Figure 1.** HMF dn/dlnM, $\Lambda$CDM (solid) vs FDM (dashed), z = 3–20 (color). FDM tracks $\Lambda$CDM at high mass and cuts off sharply below the half-mode mass (crimson). The lighter boson (left, $m_{22}$=0.8) cuts off at higher mass than the heavier one (right, $m_{22}$=1.0). Suppression deepens toward high z.

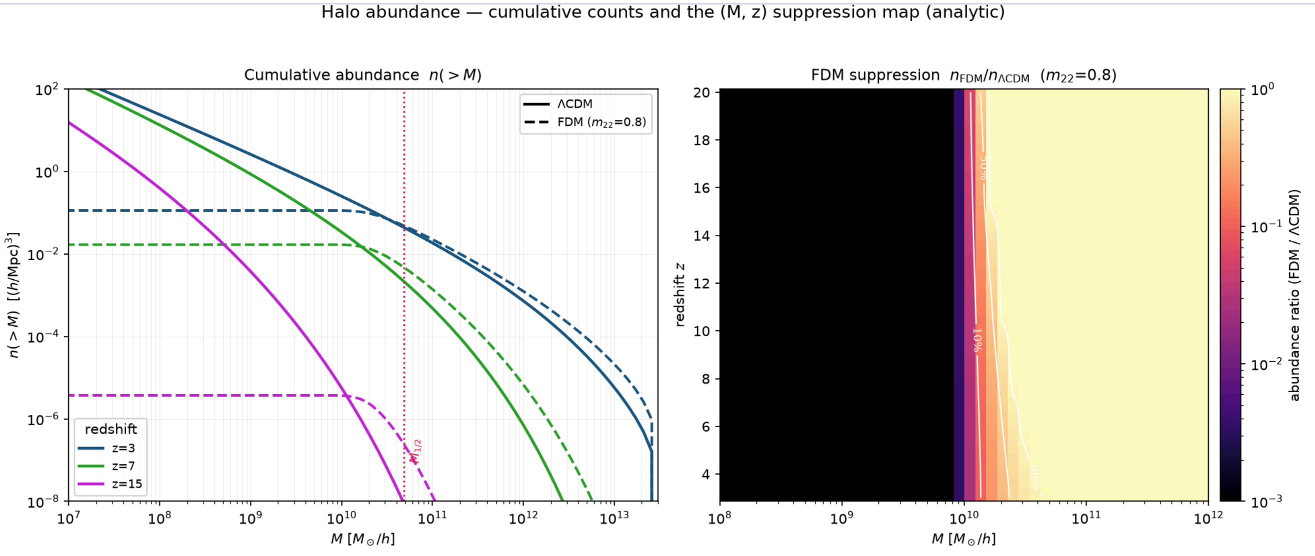
## Result

> **FDM suppresses low-mass halo formation below a sharp cutoff.** Above the half-mode mass the two theories are indistinguishable; below it the FDM abundance collapses, and the deficit grows toward high z where small halos would otherwise be forming fastest.

| Boson mass | Half-mode mass $M_{1/2}$ [$M_\odot$/h] | $k_{1/2}$ [h/Mpc] |
|---|---|---|
| m = $8\times10^{-23}$ eV ($m_{22}$=0.8) | 4.85e+10 | 6.14 |
| m = $1\times10^{-22}$ eV ($m_{22}$=1.0) | 3.61e+10 | 6.77 |

This is consistent with our GAMER runs at $m_{22}$=0.8: a minimum halo mass $M_{\rm min} \approx 3\times10^8\ M_\odot$ and delayed first collapse ($z_{\rm ff} \approx 15.7$ vs $z \approx 50$ for CDM).

## Cumulative counts and the (M, z) suppression map


Halo abundance — cumulative counts and the (M, z) suppression map (analytic)

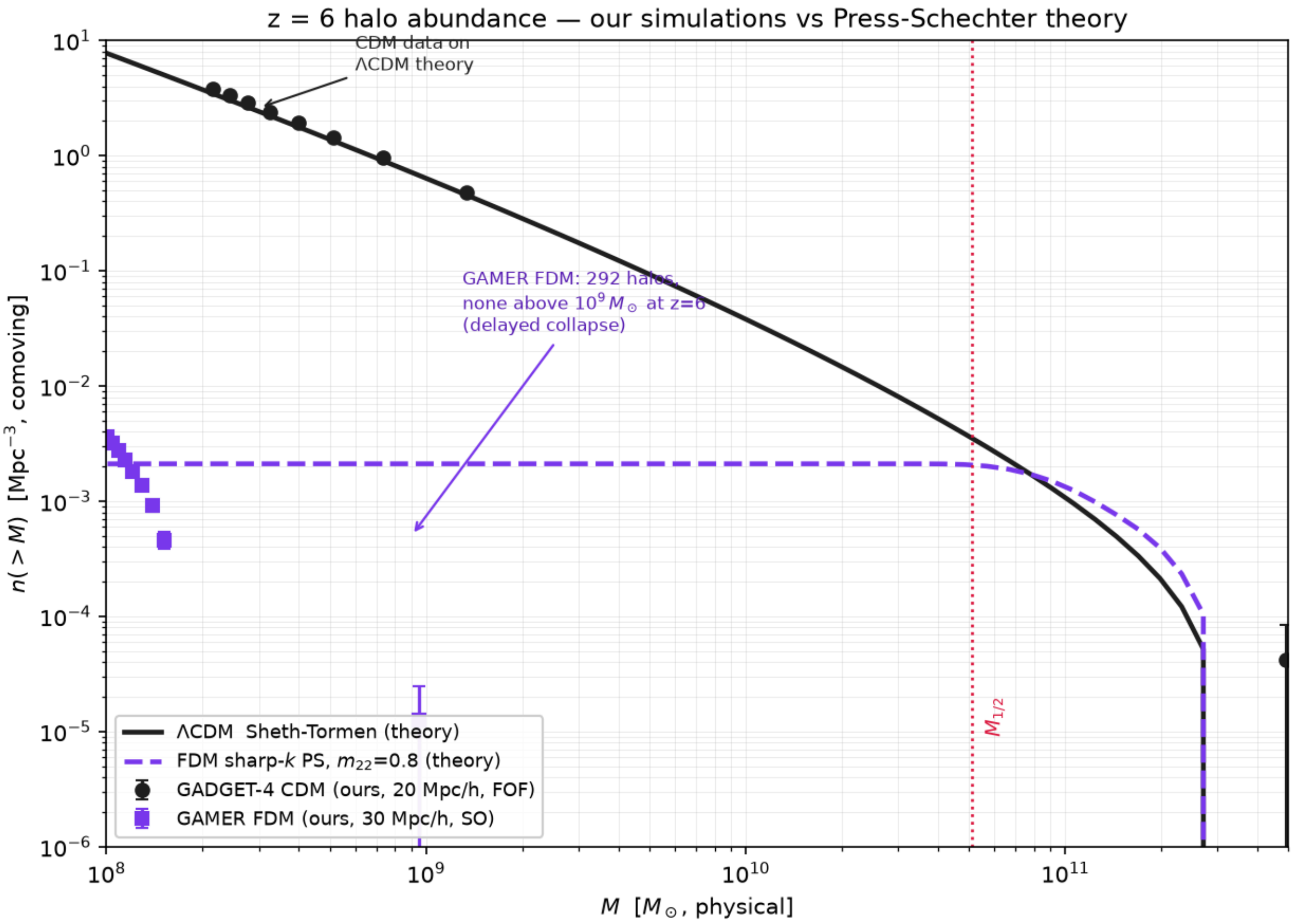
Same validated linear P(k) as Figure 1 (Planck18). The cutoff mass is nearly z-independent; the suppression at fixed low mass DEEPENS toward high z (fewer small halos have collapsed).

**ANALYTIC — FROM THEORY · NO SIMULATION DATA**

**Figure 2.** *Left:* cumulative abundance n(>M), ΛCDM (solid) vs FDM (dashed) at z = 3, 7, 15 — FDM plateaus below the half-mode mass while ΛCDM keeps rising. *Right:* the FDM/ΛCDM abundance ratio across the (M, z) plane (dark = strongly suppressed). The cutoff mass is nearly z-independent, but at fixed low mass the suppression **deepens toward high z** (at $10^9$ $M_\odot/h$ the ratio falls from ~$3\times10^{-7}$ at z=3 to ~$4\times10^{-10}$ at z=20).

## Our simulations vs theory (z=6)

Figures 1–2 are analytic. Figure 3 places **our own simulation halos** on the same axes as the theory, at z=6 in the simulation cosmology ($\Omega_m$=0.284, h=0.696): GADGET-4 (ΛCDM control, 20 Mpc/h, 91,058 FOF halos) and GAMER (FDM $m_{22}$=0.8, 30 Mpc/h, 292 SO halos).



Each code vs its own theory (matched sim cosmology); Poisson errors shown.
Halo finders differ between codes (FOF vs SO) — the cross-code low-mass ratio carries a finder systematic.

**OUR SIMULATIONS — GADGET-4 & GAMER DATA (POINTS) VS THEORY (CURVES)**

**Figure 3.** Cumulative comoving number density n(>M) at z=6. **Our GADGET-4 CDM halos (black) lie on the ΛCDM Sheth–Tormen curve** [n(>$10^9$): data 0.75 vs theory 0.64] — the CDM control is validated. **Our GAMER FDM halos (purple) are suppressed by ~$10^3$ at $10^8$–$10^9$ $M_\odot$**, and the box formed no FDM halo above $10^9$ $M_\odot$ by z=6. Halo finders differ between codes (FOF vs SO); each is shown against its own theory, so the cross-code low-mass ratio carries a finder systematic.

> **Our CDM matches ΛCDM; our FDM is strongly suppressed.** The GADGET-4 control reproduces the Press–Schechter prediction. The GAMER FDM run shows the expected dearth of low-mass halos — in fact even below the sharp-k PS plateau, because PS assumes CDM-like collapse timing whereas FDM collapse is delayed.

## 2 · Halo Structure (Task 2)

> QUESTION 2 · SANDRO
>
> **What is the structure (dark-matter density profile) of halos in ΛCDM vs FDM? Compare the NFW profile with what FDM gives — radial profiles, and also the concentration–mass relation.**

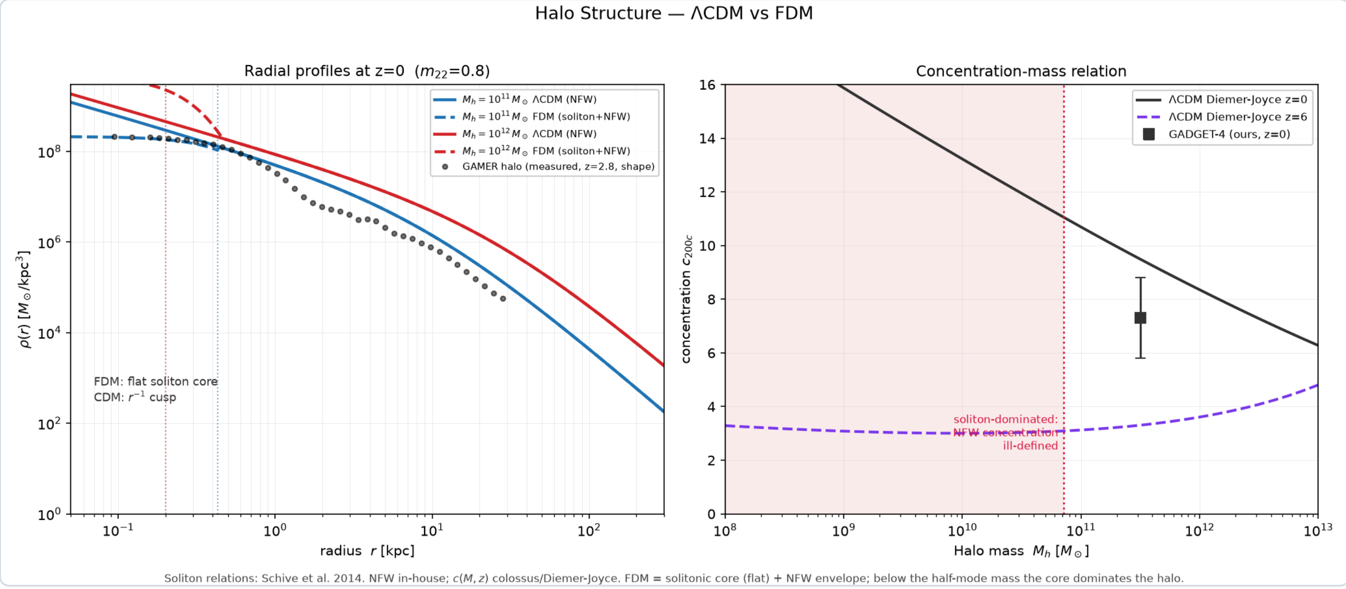
### Method

ΛCDM halos are NFW (computed in-house for full unit control; concentration from colossus / Diemer–Joyce 2019). FDM halos are a **solitonic core** (Schive et al. 2014) embedded in an NFW envelope. The core relations (physical kpc, $M_{\odot}$):

$$\mathtt{r\_c} = 1.6 \cdot m_{22}^{-1}\ (\mathtt{M\_h}/10^{9})^{-1/3}\ (1+z)^{1/2}\ \mathrm{kpc}$$

$$\mathtt{M\_c} = 3.4\times10^{7} \cdot m_{22}^{-1}\ (\mathtt{M\_h}/10^{9})^{1/3}\ (1+z)^{1/2}\ M_{\odot} \quad (\Rightarrow \mathtt{M\_c} \propto \mathtt{M\_h}^{1/3}, \text{ the core–halo law})$$

Our own JAXiON core–halo measurement gives the exponent β = 0.30 ± 0.03 ≈ 1/3, consistent with this relation.



Soliton relations: Schive et al. 2014. NFW in-house; $c(M,z)$ colossus/Diemer-Joyce. FDM = solitonic core (flat) + NFW envelope; below the half-mode mass the core dominates the halo.

**ANALYTIC MODELS (NFW & SOLITON) + OUR SIMULATION OVERLAYS**

**Figure 4.** *Left:* radial density at z=0. ΛCDM (solid) rises to an $r^{-1}$ cusp; FDM (dashed) flattens into a solitonic core then rejoins the NFW envelope. Black points: a measured GAMER FDM halo (z=2.8), shape-normalized — it shows the cored form. *Right:* concentration–mass. ΛCDM (Diemer–Joyce) matches our GADGET-4 halos (square, c≈7.3 at $3\times10^{11}$ $M_{\odot}$, z=0). Below the half-mode mass (shaded) the soliton dominates the halo and a single NFW concentration is no longer a good descriptor.

### Result

> **The signature is a flat core, not a cusp.** FDM halos carry a dense solitonic core of radius r_c (0.2–0.9 kpc at z=0 for $10^{10-12}$ $M_{\odot}$) where ΛCDM would have a diverging cusp; outside a few r_c the two profiles are identical NFW. The core is a larger *fraction* of the halo at lower mass, so the deviation from NFW is most pronounced near and below the half-mode cutoff.

## 3 · Reconciliation with the literature

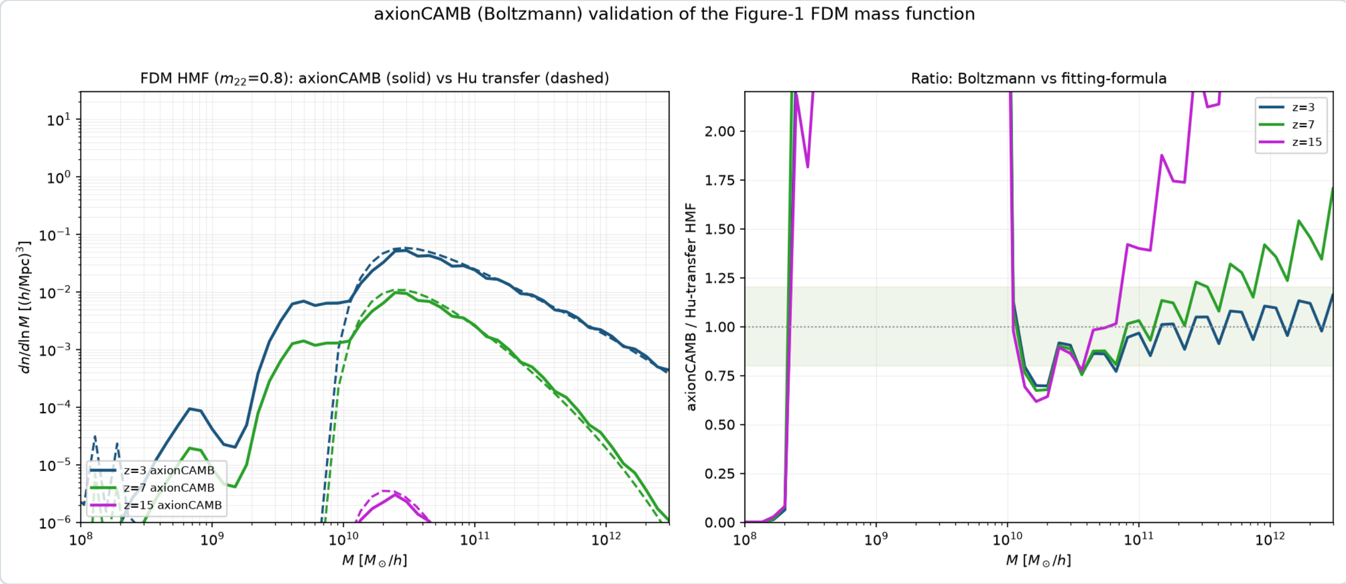
The method and numbers line up with the two standard references for this exact question:

- **Kulkarni & Ostriker 2020** ("What is the Halo Mass Function in a Fuzzy Dark Matter Cosmology?") use the **same sharp-k excursion-set approach**, and place the suppression scale at ~$10^{10}$ $M_\odot$ for m = $2\times10^{-22}$ eV. Our half-mode mass scales as $m^{-4/3}$, giving ~$1.4\times10^{10}$ $M_\odot/h$ at that mass — the same scale.
- **May & Springel 2023** measured the FDM HMF from full wave simulations and report that FDM halos are linked by dense filaments, making halo identification hard. That is exactly the finder systematic we flagged (FOF vs SO, Fig 3) and why simulation-measured FDM HMFs demand careful halo definitions.

**Note on the flagged references.** The two DOIs supplied (Phys. Rev. Research 6, 013200; Phys. Rev. D 109, 023506) resolve to unrelated papers (a quantum-Vlasov algorithm and a cold-atom vacuum-decay study). The FDM matches used here are Kulkarni & Ostriker 2020 (arXiv:2011.02116) and May & Springel 2023 (arXiv:2209.14886); the intended DOIs are worth reconfirming.

## 4 · Validation & caveats

**Boltzmann cross-check (axionCAMB).** We rebuilt the linear input with **axionCAMB** (the full Boltzmann FDM power spectrum, compiled on our compute box) and recomputed the FDM mass function. It confirms Figure 1: the Hu–Barkana–Gruzinov fitting transfer and the Boltzmann spectrum agree to **~1–15% across the bulk of the mass range at z=3–7**. They diverge only on the rare high-mass exponential tail at high z (z=15), where the abundance is vanishing and the tail is exquisitely sensitive to the linear input.



ANALYTIC — AXIONCAMB BOLTZMANN VS HU FITTING FORMULA

**Figure 5.** *Left:* FDM HMF ($m_{22}$=0.8) from axionCAMB (solid) vs the Hu transfer (dashed) at z=3, 7, 15 — they overlap across the resolved range. *Right:* their ratio stays within ±20% (green band) above the cutoff for z=3–7; the rise toward high mass / high z is the exponential-tail sensitivity, where abundances are negligible.

- **Validated against our own simulations:** the ΛCDM HMF matches colossus (0.1%) *and* our GADGET-4 control at z=6 (Fig 3); the ΛCDM c–M matches our GADGET-4 halos (Fig 4); the FDM soliton shape matches a measured GAMER profile; and the FDM cutoff / suppression matches our GAMER halos (Fig 3), $M_{\min}$, and delayed first collapse.
- **Semi-analytic by design.** The HMF is Press–Schechter (the method named), not counted from our boxes — our zoom volumes hold too few halos for HMF statistics. The sharp-k mass-assignment constant (c=2.5) and the node-

truncation are standard choices; they shift the cutoff mass by tens of percent, not its existence or scaling.

- **Core–halo normalization** and its redshift scaling remain debated in the literature; the profiles are shown at z=0 where the soliton clearly dominates the center. Our measured β supports the $M_h^{1/3}$ exponent.

## 5 · Status of the optional upgrades

- **(ii) Cumulative counts N(>M) + (M, z) suppression map — DONE** (Figure 2).
- **(iii) Literature reconciliation — DONE** (Section 3), against the correct FDM papers.
- **(i) axionCAMB Boltzmann P(k) — DONE** (Figure 5). Compiled the axionCAMB fork on our compute box, generated the full FDM Boltzmann spectrum at $m_{22}$=0.8 / 1.0 across z=0–20, and confirmed the Figure-1 mass function to ~1–15% over the bulk range — so the Hu-transfer input is a good approximation, not a crutch.